Fundamentals of Engineering Review for Dynamics

(retired)

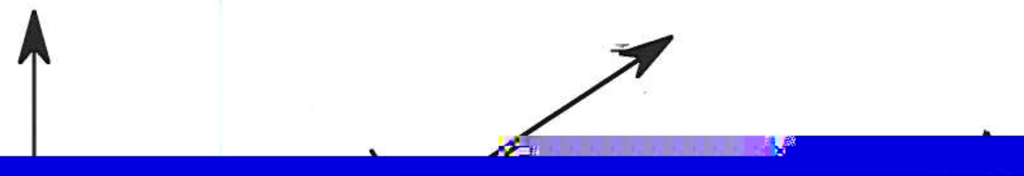
Review Notes available in PDF format @

u/eng/docs/FE-Exam-Review/Dynamics.p f

- I

$\underline{P}$ position orien tion

$),(\quad,\quad),(\ _n\ a_t$



D
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t

ds

v = 0

d

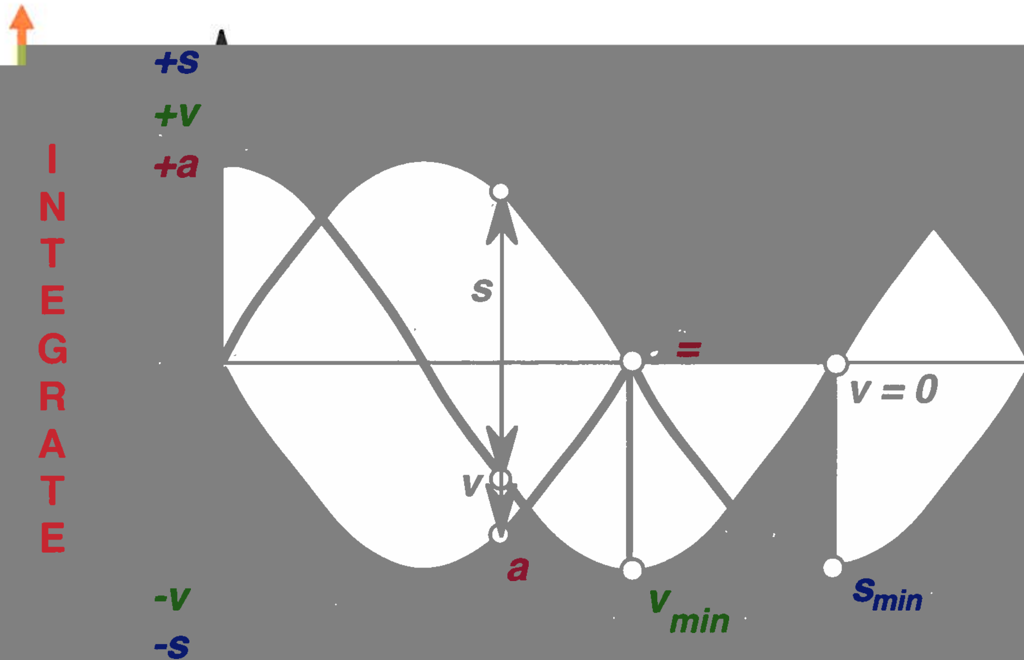
v

t

- *Typica*
  - Polynomia ,   g

Rectilinear M

- Positio
- S ( )



a *a*

a

s *a*

Rectilinear Kinematics: Accel. a function of velocity – a(v)

- Substituting BC's helps resolve the unknown constant *k*

p *p* the ship as a function of
s

$$v_f = v(t) = \frac{8}{6t+1} \quad (knots)$$

From here, there are two alternatives for resolving the second question

and as was seen before

$$s(t = 1/6) \Rightarrow s(v = 4) = \underline{\underline{\frac{4}{3}\ \ln(2)}} \quad ($$

Q.E.D.

2D Curvilinear Kine

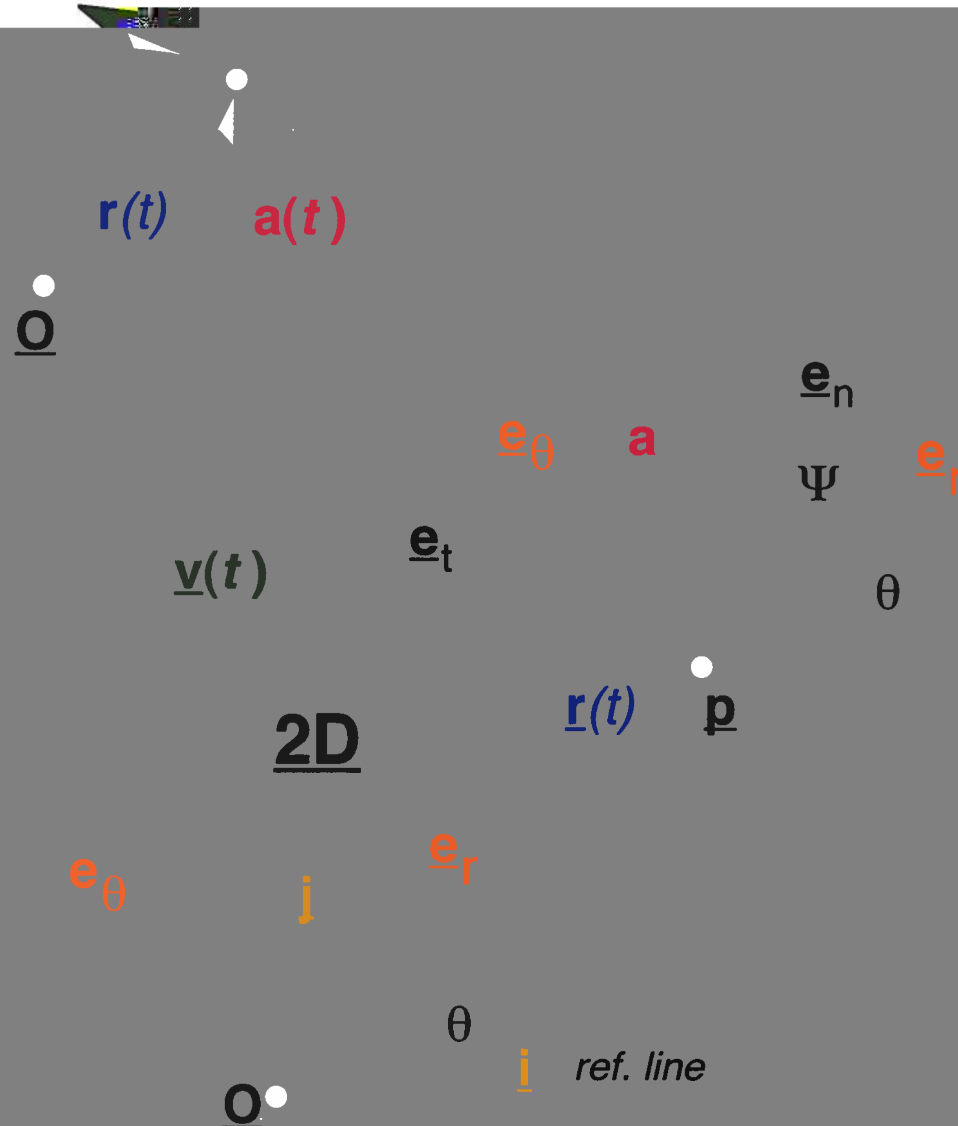
- **Position**

$$\underline{\mathbf{r}}(t) = x(t)\underline{\mathbf{i}} + y(t)\underline{\mathbf{j}}$$
$$= r(t)\underline{\mathbf{e}}_r$$
$$?\,path\,?$$

- **Velocity**

$$\underline{\mathbf{v}}(t) = \dot{\underline{\mathbf{r}}}(t) = \dot{x}\,\underline{\mathbf{i}} + \dot{y}\,\underline{\mathbf{j}}$$
$$= v\underline{\mathbf{e}}_t = \dot{s}\,\underline{\mathbf{e}}_t$$
$$= \dot{r}\,\underline{\mathbf{e}}_r + r\dot{\theta}\,\underline{\mathbf{e}}_\theta$$

- **Acceleration**



- Cartesian <-> Polar <-> Path

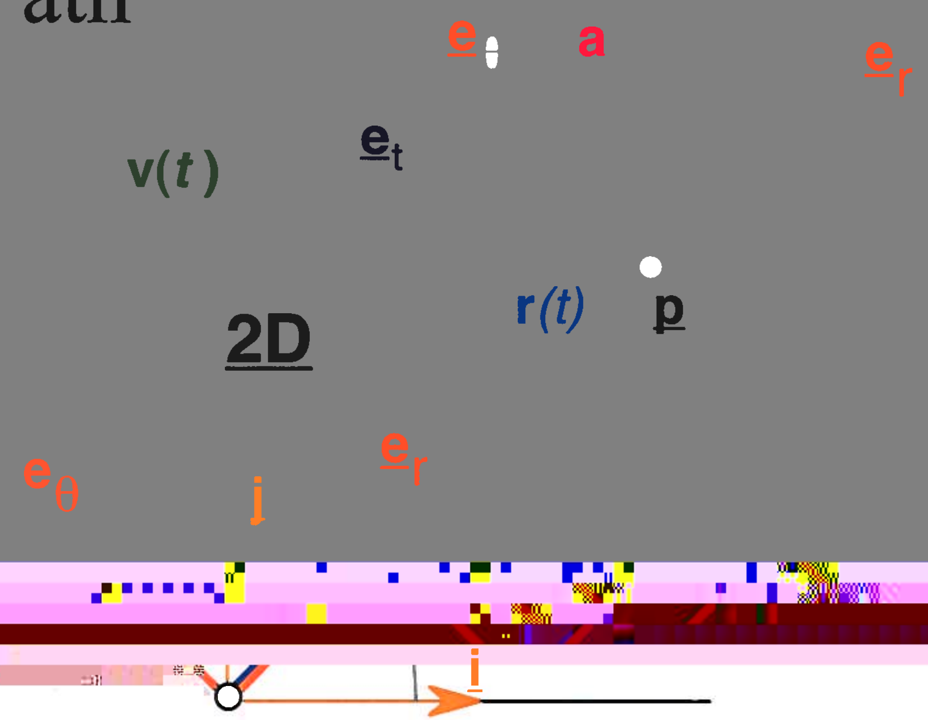
$$\underline{\mathbf{r}}(t) = x\underline{\mathbf{i}} + y\underline{\mathbf{j}} = r\underline{\mathbf{e}}_r$$

$$\underline{\mathbf{e}}_r = \cos\theta\underline{\mathbf{i}} + \sin\theta\underline{\mathbf{j}} = \frac{x}{r}\underline{\mathbf{i}} + \frac{y}{r}\underline{\mathbf{j}}$$

$$\underline{\mathbf{e}}_\theta = \underline{\mathbf{k}} \times \underline{\mathbf{e}}_r = \cos\theta\underline{\mathbf{j}} - \sin\theta\underline{\mathbf{i}}$$

$$\underline{\mathbf{i}} = \cos\theta\underline{\mathbf{e}}_r - \sin\theta\underline{\mathbf{e}}_\theta$$

$$\underline{\mathbf{j}} = \underline{\mathbf{k}} \times \underline{\mathbf{i}} = \cos\theta\underline{\mathbf{e}}_\theta + \sin\theta\underline{\mathbf{e}}_r$$



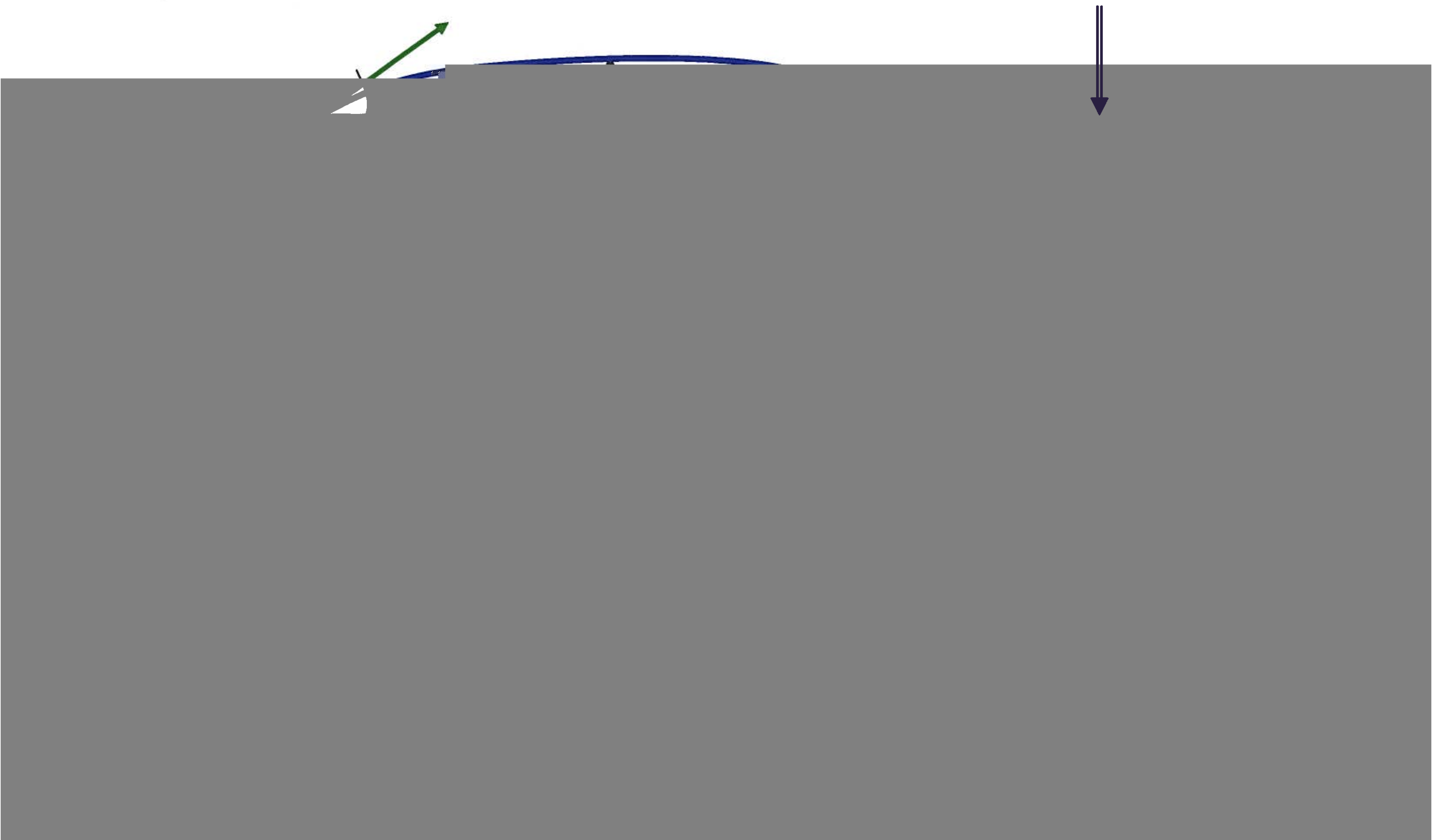
$$\underline{\mathbf{v}}(t) = \dot{\underline{\mathbf{r}}}(t) = \dot{x}\underline{\mathbf{i}} + \dot{y}\underline{\mathbf{j}} = v\underline{\mathbf{e}}_t = \dot{s}\underline{\mathbf{e}}_t$$

$$\underline{\mathbf{e}}_t = \frac{\underline{\mathbf{v}}}{|\underline{\mathbf{v}}|} = \frac{\dot{x}}{v}\underline{\mathbf{i}} + \frac{\dot{y}}{v}\underline{\mathbf{j}}$$

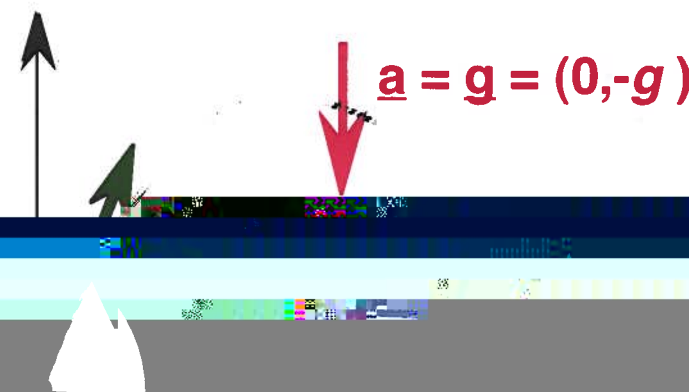
$${}_n + \sin\Psi\underline{\mathbf{e}}_t$$

$$\underline{\mathbf{e}}_n = \underline{\mathbf{k}} \times \underline{\mathbf{e}}_t = \frac{\dot{x}}{v}\underline{\mathbf{j}} - \frac{\dot{y}}{v}\underline{\mathbf{i}}$$

$\underline{a} = \underline{g}$



$\underline{a} = \underline{g} = (0,-g)$


a = g = (0,-g)

$$\underline{a} = \underline{g} = (0, -g)$$

15°

20 km/hr

$\underline{a} = \underline{g} = (6,-9)$

l &Kraige 2-8

$\underline{e}_n$

20

1 ity

$\underline{a} = \underline{g} = (6,-9)$

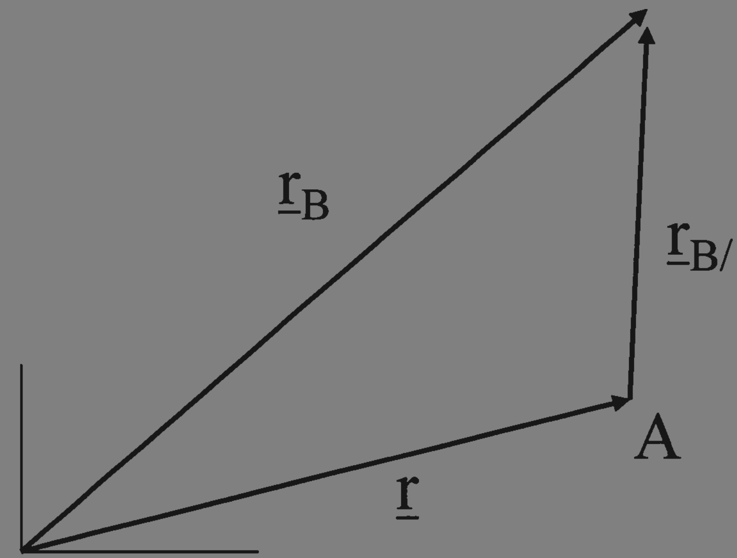
$\underline{O}$

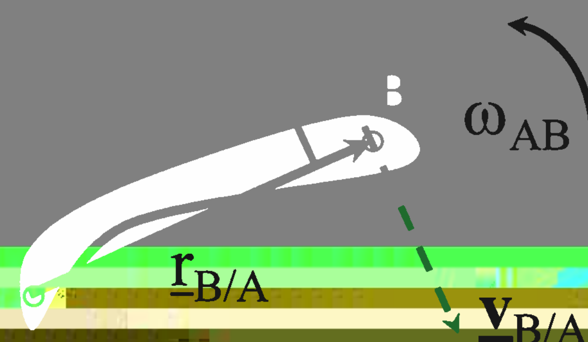
RELATIVE MOTION

$\underline{\mathbf{r}}_B = \underline{\mathbf{r}} \quad + \underline{\mathbf{r}}_{B/}$

$\underline{\mathbf{v}}_B = \underline{\mathbf{v}}_A + \underline{\mathbf{v}}_{B/}$

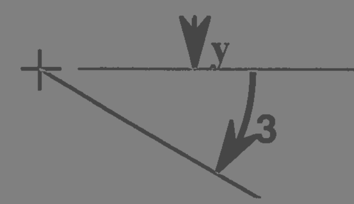
$\underline{\mathbf{a}}_B = \underline{\mathbf{a}} \quad + \underline{\mathbf{a}}_{B/}$

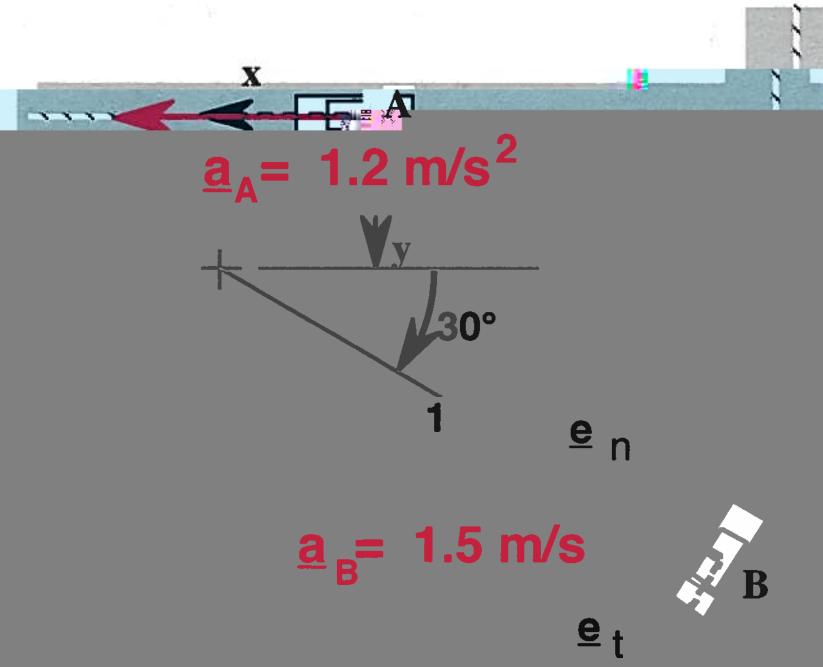




A A A B

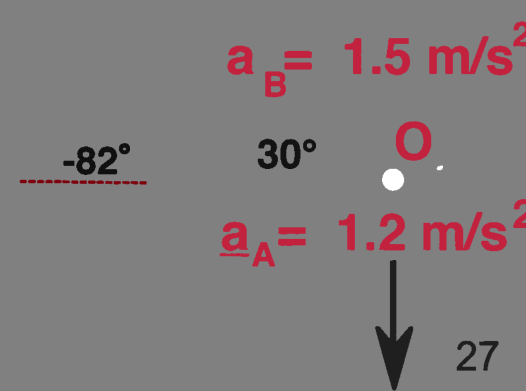
A





30°j

0°j + sin 30°i



Acceleration Polygon (Graphical)

$$\underline{\mathbf{a}}_B = \underline{\mathbf{a}}_A + \underline{\mathbf{a}}_{B/A}$$

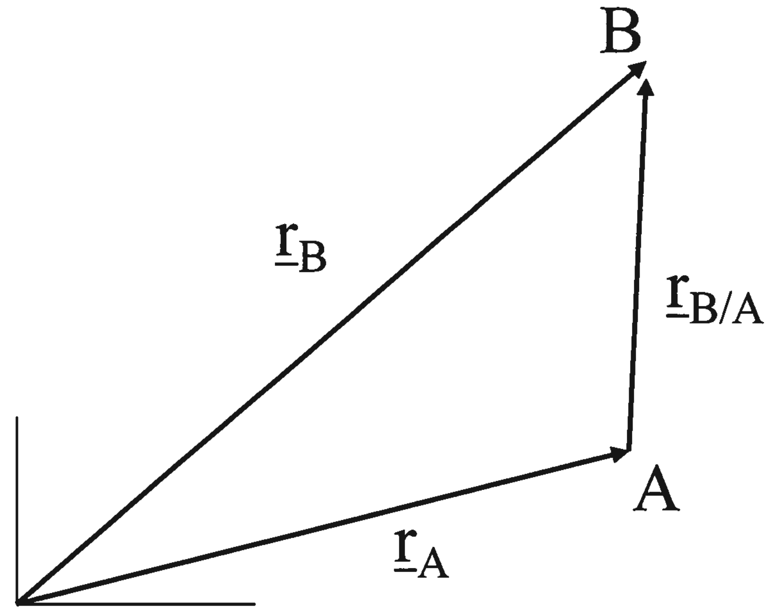
$$\Rightarrow \underline{\mathbf{a}}_{B/A} = \underline{\mathbf{a}}_B - \underline{\mathbf{a}}_A$$

$\underline{\mathbf{r}}_{B/C} = \underline{\mathbf{r}}_{B} - \underline{\mathbf{r}}_{C}$ *not tr e c*

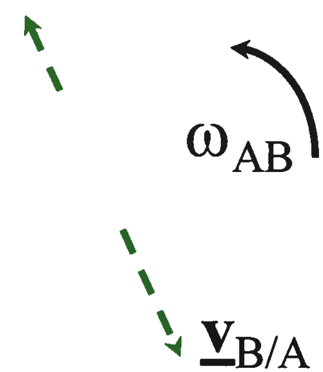
$\underline{\mathbf{v}}_{B C} = \underline{\mathbf{v}}_{B} - \underline{\mathbf{v}}_{C}$

$\dot{r}_{B C}$ *not true scale!*

(

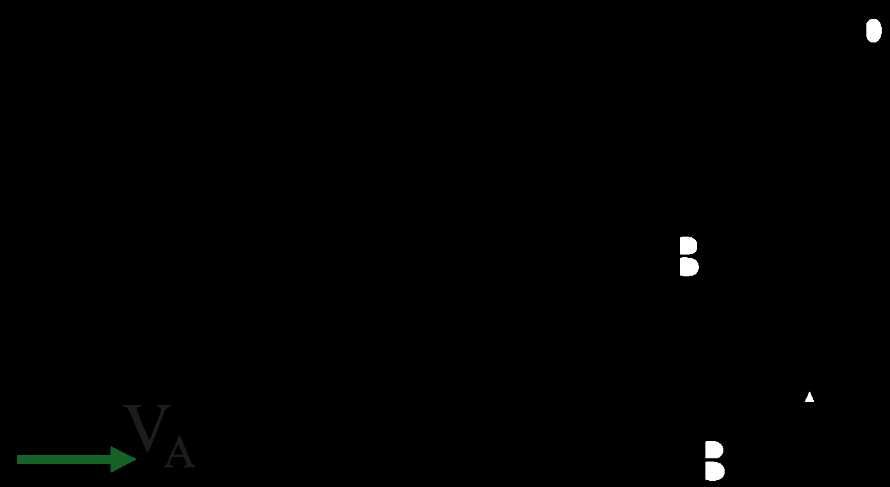


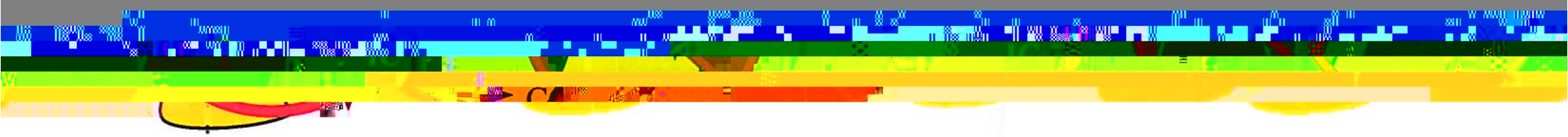
B
r_B
r_B/A
A
r_A



ω_AB
v_B/A

wo points on a rigid body:





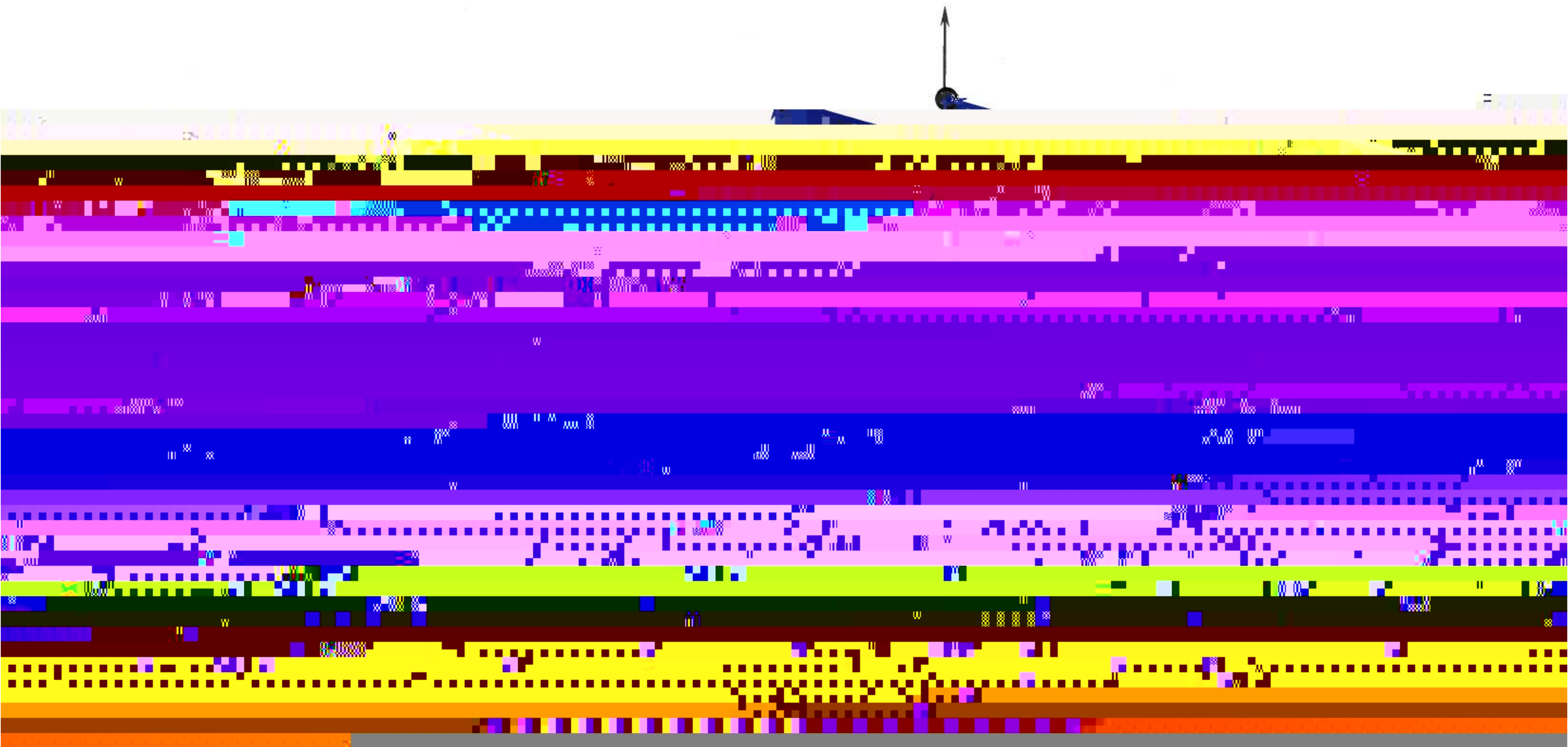
S

s f

=

B

ω ω ω

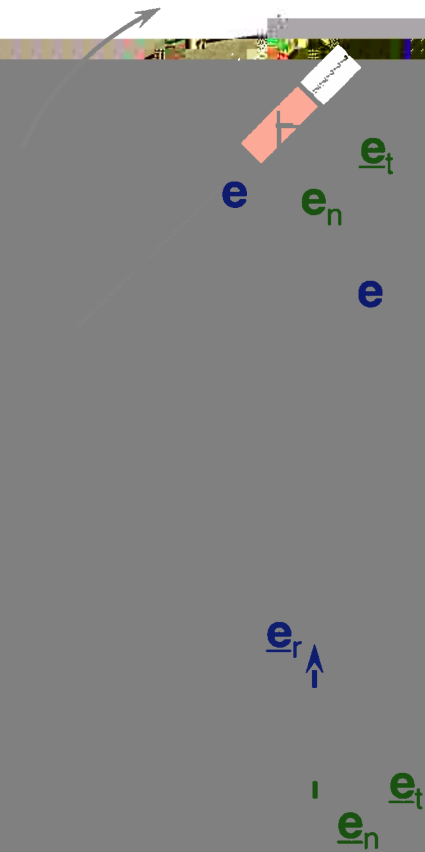


Newtons Law

$\underline{\mathbf{a}}_{CG}$

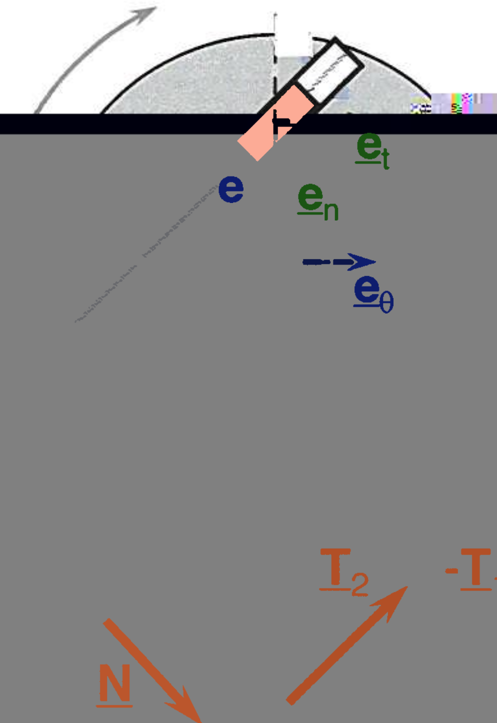
$\underline{\mathbf{a}}_{CG}$


CG

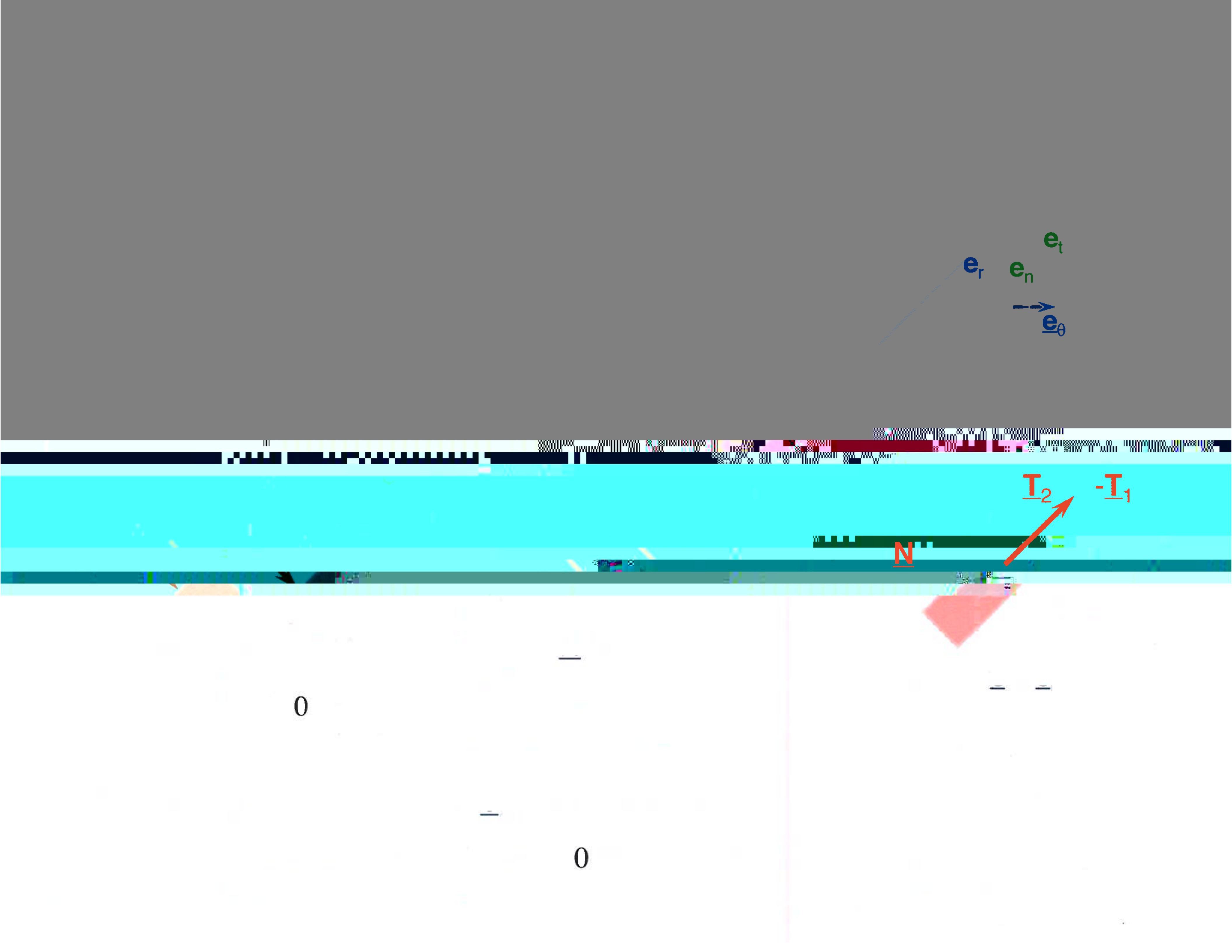


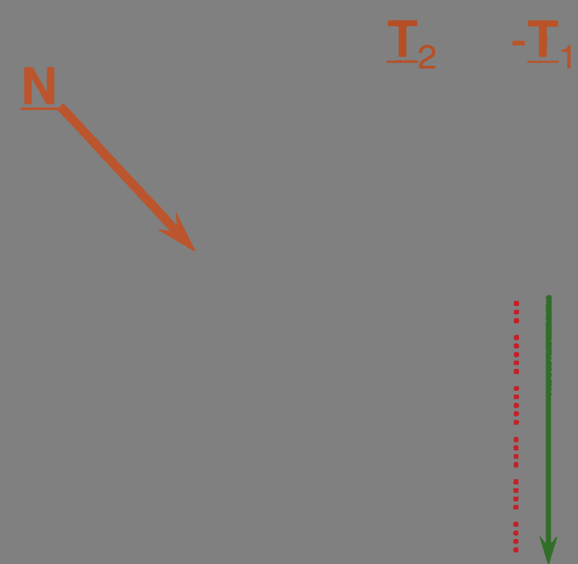
$\underline{e}_n$ $\underline{e}_t$

$\underline{e}_r$ $\underline{e}_\theta$

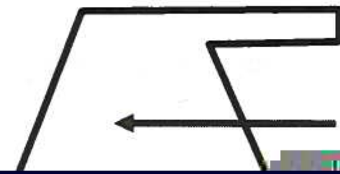


e
e_t
e_n
e_θ
T_2
-T_1
N

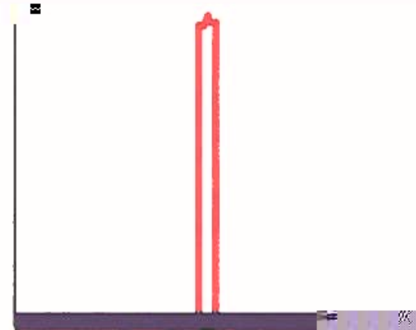

e_t
e_r
e_n
e_θ
T_2
$-T_1$
N
0
0

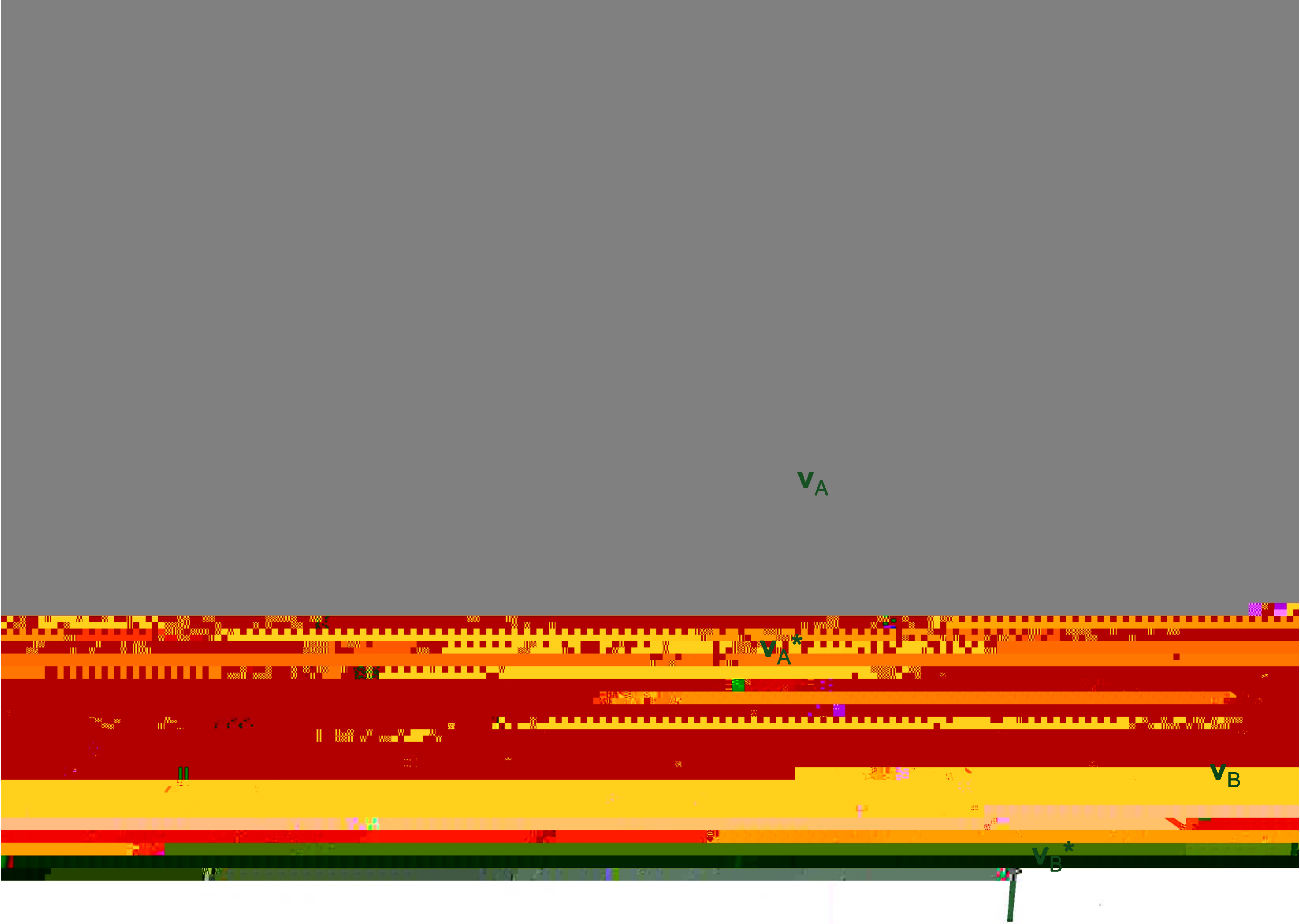

$\underline{T}_2$
$-\underline{T}_1$
$\underline{N}$

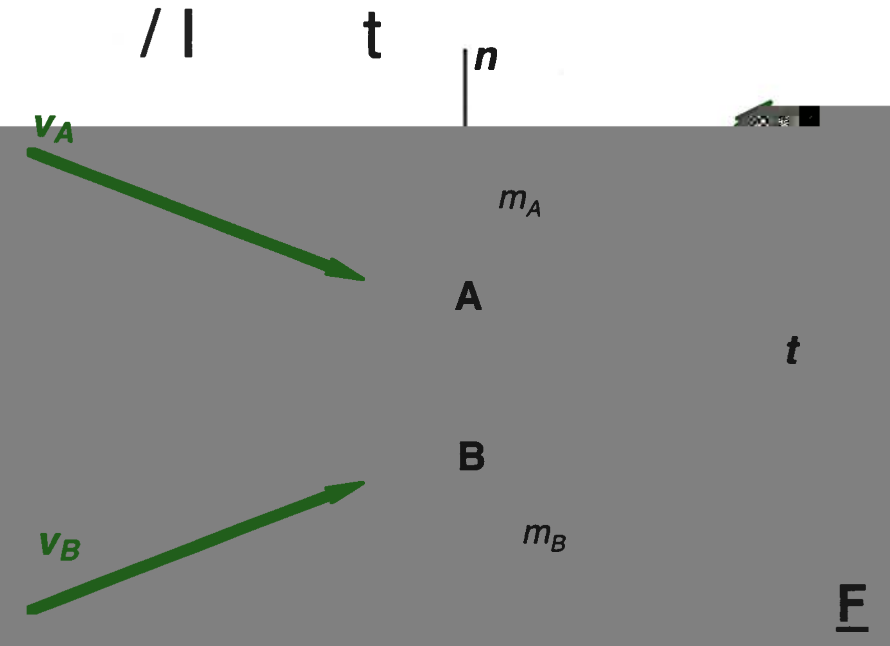
Time




B1
B2




$\mathbf{v}_A$
$\mathbf{v}_A^*$
$\mathbf{v}_B$
$\mathbf{v}_B^*$

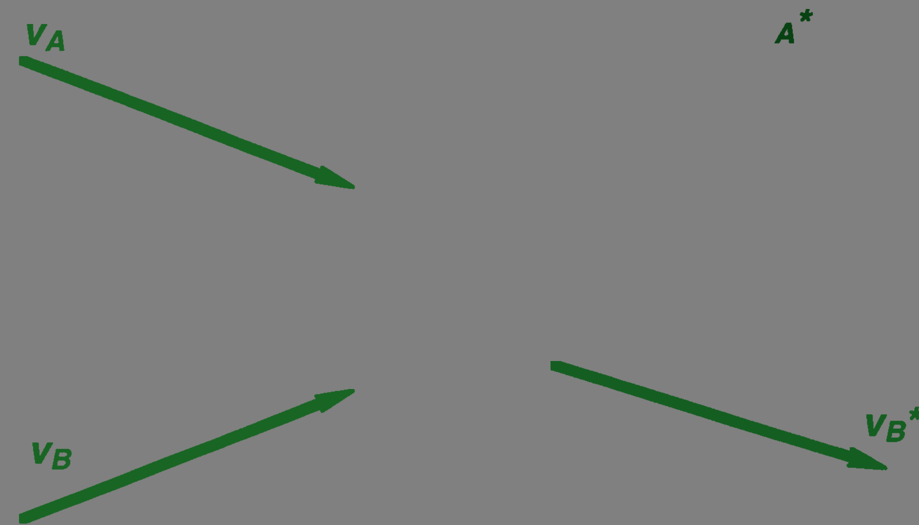


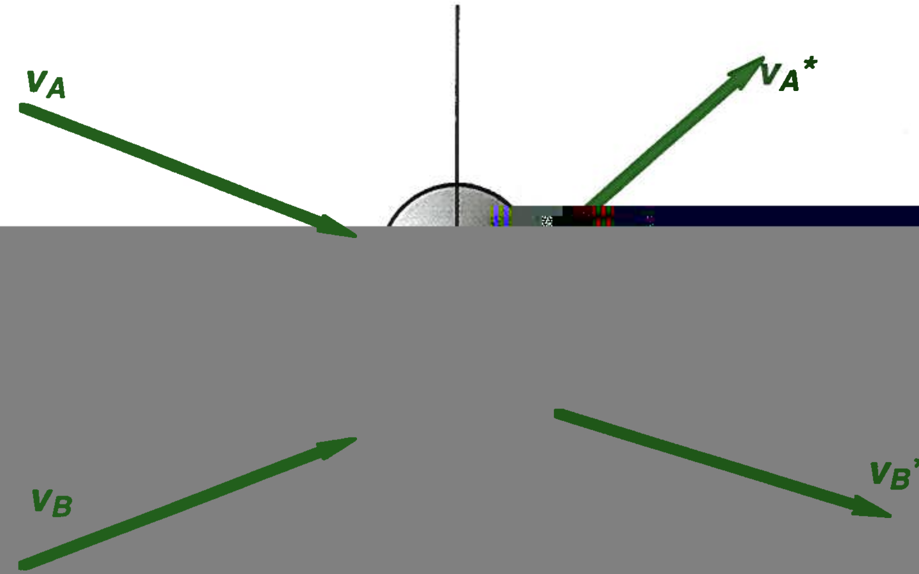
(3) Coefficient of Restitution: Rel. Velocities along Common NORMAL!

F

$$e = \left.\frac{v_{Relative\ Separation}}{v_{Relative\ Approach}}\right|_{Normal} = \frac{v_{Bn}^{*} - v_{An}^{*}}{v_{An} - v_{Bn}}$$

(Perfectly Plastic) $0 \le e \le 1$ (Perfectly Elastic)


v_A
A^*
v_B
v_B^*



v_A
v_A^*
v_B
v_B^*

WORK-ENERGY for Rigid Bodies

ROTATION

ROTATION

mg

otion (2D)

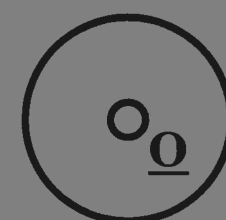
k_{spr}

m_{50} k_0

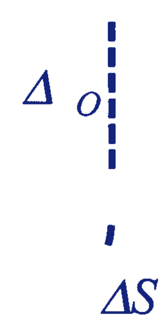
O y

m x

FBD



O


Δ
O
ΔS

mg

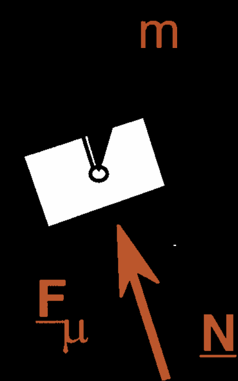
F_s

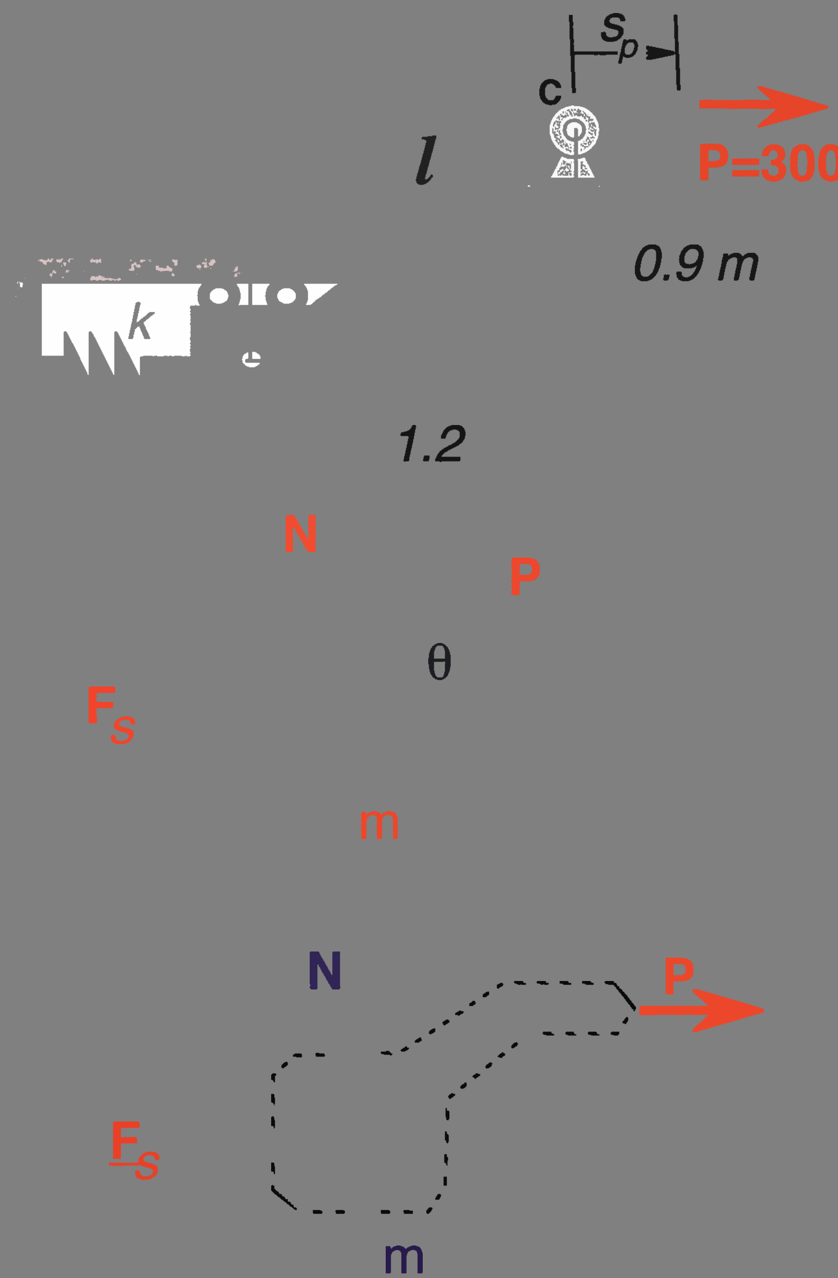
N

g

in

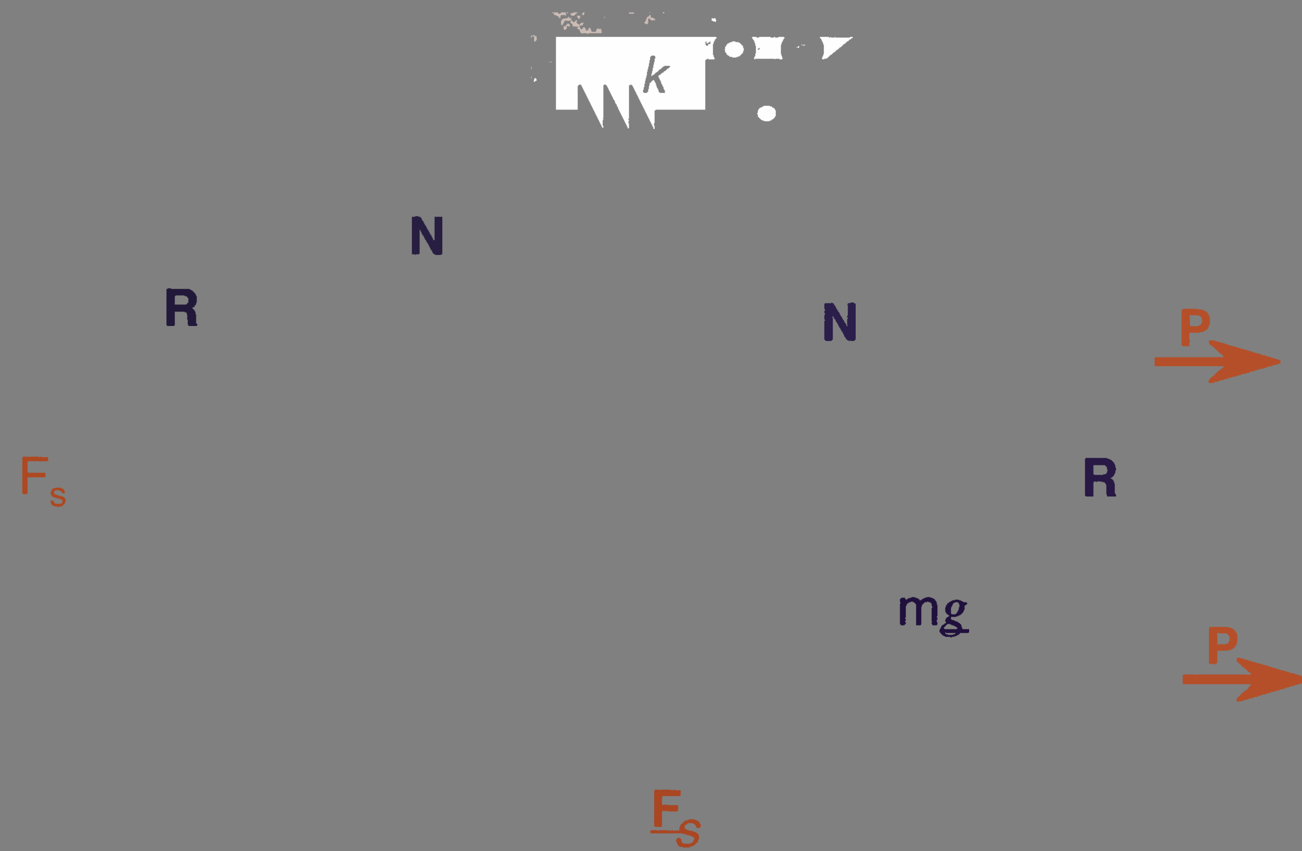
direction of displacement

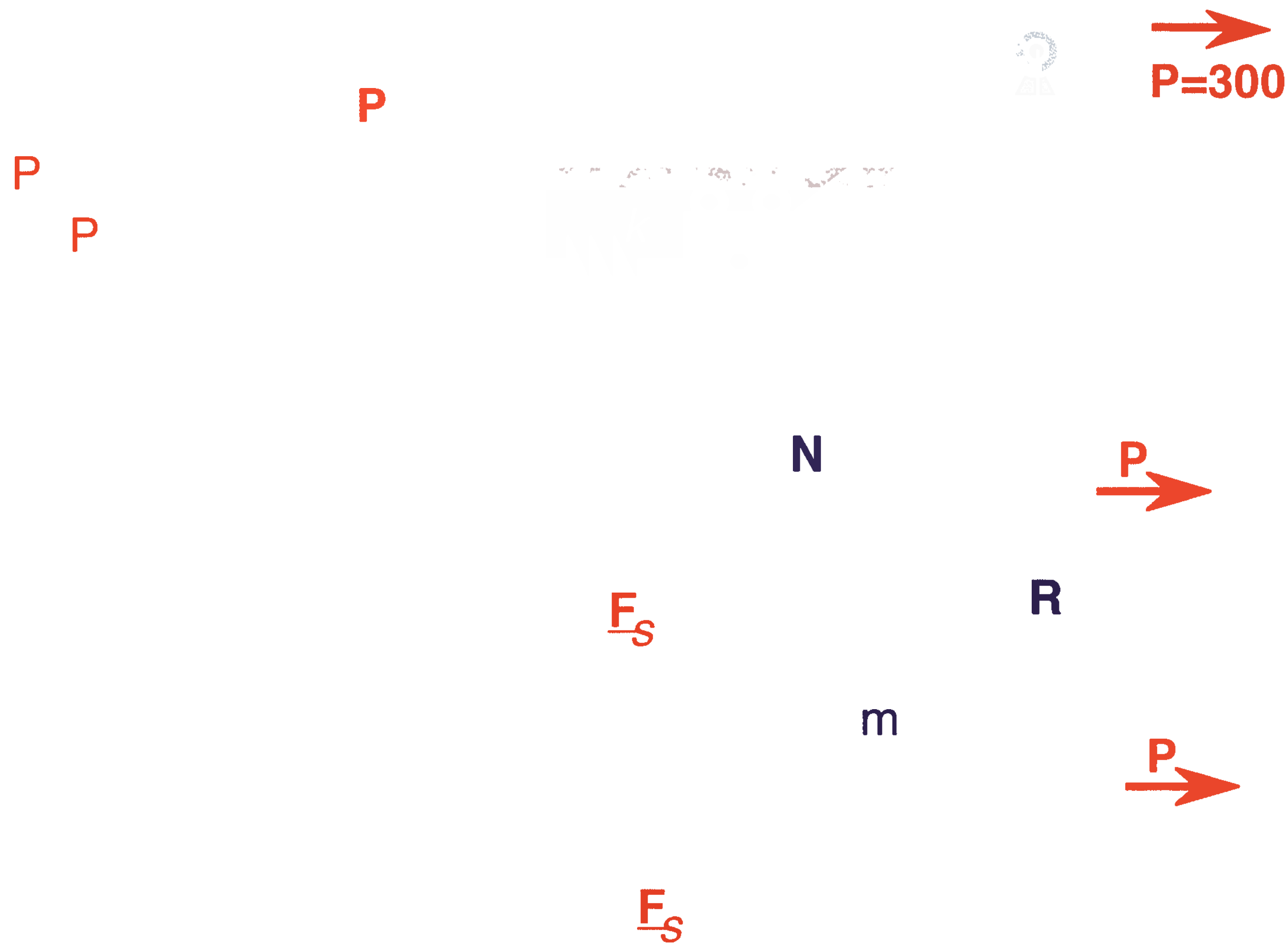




ACTIVE Force Diagram!

ACTIVE Force Diagram!




P=300
P
P
P
N
P
F_s
R
m
P
F_s

A A

6 +

E

A