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1. An insulated disk of radius " a " and infinitesimal thickness carries a positive definite surface charge density " σ ".
 - a) Calculate the electrostatic potential at the center of the disk.
 - b) Calculate the electrostatic potential at an arbitrary point on the axis of symmetry.
 - c) Calculate the electrostatic potential at the edge of the disk.
 - d) Using the results derived above, deduce the direction of the electric field in the plane of the disk.

 2. Consider a non-relativistic particle of mass " m " and charge " e " that is acted on by an external force, " \vec{F} ". The instantaneous power radiated by the charge is given by the Larmor formula
 - a) Write down the relativistic generalization of the Larmor Formula.
 - b) Express your answer to part a) in terms of the external force \vec{F} , assuming that the charged particle is being accelerated in a linear accelerator.
 - c) In typical linear accelerators the energy gains are approximately 10^6 eV per meter. Demonstrate that for relativistic particles the radiation losses are negligible in comparison with the energy gains.

3. Consider a simple model for a dielectric in which the atomic electrons are bound to a fixed site by means of a harmonic restoring force $-k\vec{x}$. Suppose that in addition the electrons are acted upon by a damping force $-\gamma_0\dot{\vec{x}}$.

$\frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$

7. The plates of a semi-infinite capacitor are maintained at a constant potential difference V by means of a battery. The plates are a distance d apart. A charge q of mass m is released from rest at the surface of one plate and moves toward the other plate under the influence of the electric field. Ignoring complicating effects, such as gravity and image charges,
- Derive an expression for the power radiated by the charge during its motion.
 - Calculate the total energy radiated by the charge during its motion and compare with the change in kinetic energy.
 - Identify any assumptions that underlie your calculation.

NOTE: The results in (a) and (b) should be expressed in terms of q , m , V , d and c (velocity of light).

8. Consider a general current distribution $\vec{j}(\vec{r}, t)$ localized in a small region of space.
- Define the associated magnetic moment $\vec{\mu}$.
 - Write down a general result for the vector potential $\vec{A}(\vec{r}, t)$ in terms of $\vec{j}(\vec{r}', t')$ (1 point) and use it to write down (proof not required) an expression for $\vec{A}(\vec{r}, t)$ in terms of $\vec{\mu}$ in the case where $|\vec{r}| \gg |\vec{r}'|$.
 - Assuming that a magnetostatic field is due entirely to a localized distribution of permanent magnetization, show that

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{3\vec{r}(\vec{r} \cdot \vec{\mu}) - \vec{r}^2 \vec{\mu}}{r^5}$$

where the integral is taken over all space

9. A plane polarized electromagnetic wave of frequency ω in free space is incident normally on the flat surface of a nonpermeable medium of conductivity σ and dielectric constant ϵ .
- Given that the ratio of the amplitudes of the reflected and incident waves is

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"2)(A charge is situated at (2a, 0, 0) and a charge ! is situated at (-a, 0, 0).

- a) Identify every point in space where the electric field E is zero.
- 3' Consider two concentric metal (conducting) spheres, of radii R_1 and R_2 carrying total charges Q_1 and Q_2 respectively. What is the potential of the outer sphere, the inner sphere and inside inner sphere .

17. The vector potential for a given current distribution with a sinusoidal time dependence $\mathbf{j}(\mathbf{r}, t) = \mathbf{j}(\mathbf{r}) e^{-i\omega t}$ is given by:

$$\mathbf{A}(\mathbf{r}, t) = \frac{\mu_0}{4\pi r} \int \mathbf{j}(\mathbf{r}', t - r/c) dV'$$

where

There are three length scales to consider:

- 1. r : size of the region where
- 2. λ :

3. : distance of the observation point from the current.

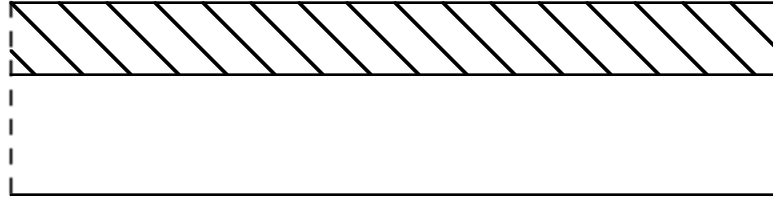
Derive the following electric dipole radiation field expressions *justifying each approximation with relevant conditions e.g. (length scale)₁ << (length scale)₂* etc.

where

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- a) Derive an expression for its capacitance.
 - b) A parallel plate capacitor is formed with two metal plates 10 cm by 10 cm separated by an air space 4 mm wide. The capacitor is charged from a 400 V supply. Then, the supply is disconnected and immediately a sheet of glass 10 cm by 10 cm by 2 mm is placed midway between the plates.
 - i) Calculate the capacitance of the capacitor with and without the glass, for which .
 - ii) Calculate the potential difference across the capacitor after inserting the glass.
 - iii) Calculate the stored energy, before and after inserting the glass plate and explain any difference.
20. The electric potential just outside of the surface of a dielectric sphere of radius a , dielectric constant ϵ_r , and fixed surface charge density σ is
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- a) Find the electric potential everywhere outside the sphere.
 - b) Find the electric potential everywhere inside the sphere.
 - c) Determine the surface charge density .
21. Answer briefly and to the point.
- a) What is Cherenkov radiation and what condition is necessary to obtain it?

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